Acta Mech 227, 955–968 (2016)
DOI 10.1007/s00707-015-1498-9

ORIGINAL PAPER

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Generalized thermoelastic diffusion in a nanoscale beam using eigenvalue approach

Received: 9 April 2015 / Published online: 28 November 2015

Abstract In this paper, the effect of mass diffusion in a thermoelastic nanoscale beam in the context of the Lord and Shulman theory is studied. The analytical solution in the Laplace domain is obtained for lateral deflection, temperature, displacement, concentration, stress and chemical potential. Both ends of the nanoscale beam are simply supported. The basic equations are written in the form of a vector-matrix differential equation in the Laplace transform domain, which is then solved by an eigenvalue approach. The results obtained are presented graphically for the effect of time and mass diffusion to display their physical meaning.

1 Introduction

There are a number of significant problems in engineering requiring thermal stress analysis. An important class of problems arises in mechanical engineering and includes the analysis of machine components subjected to high-temperature environments and large temperature variations such as in a turbine. The coupled thermoelasticity theory was formulated by Biot [1] to eliminate the paradox inherent in the classical uncoupled theory that elastic changes have no effect on the temperature. The heat equations for both theories, however, are of the diffusion type predicting infinite speeds of propagation for heat waves, contrary to physical observations. The theory of couple thermoelasticity was extended by Lord and Shulman [2] and Green and Lindsay [3] by including the thermal relaxation time in the constitutive relations. The counterparts of our problem in the contexts of the thermoelasticity theories have been considered by using analytical and numerical methods [4–17].

Diffusion can be defined as the migration of particles from regions of high concentration to regions of lower concentration. The recent interest in the study of this phenomenon is due to its many industrial applications. The theory of thermoelastic diffusion was developed by Nowacki [18–21]. In this theory, the coupled thermoelastic model is used. Recently, Sherief et al. [22] developed the theory of generalized thermoelastic diffusion that predicts finite speeds of propagation for thermoelastic and diffusive waves. Elhagary [23] presented the effects

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